

# Tensor susceptibility calculated in the hadronization process

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## Abstract

The tensor susceptibility of QCD vacuum is calculated in the global color symmetry model. The input parameters for gluon propagators are determined via the simplified equation for calculating the pion decay constant. The reason for the great discrepancy between our results and those from QCD sum rules and from chiral constituent quark model is discussed.

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The tensor susceptibility of QCD vacuum, like the quark condensate or the gluon condensate, reflects the non-perturbative aspects of the QCD vacuum directly. It is argued that, the tensor susceptibility is related to a chiral-odd spin-dependent structure function that can be measured in the polarized Drell-Yan process ([1]-[4]). The earlier estimations for the value of tensor susceptibility were obtained by QCD sum rules techniques ([5]-[8]) or from chiral constituent quark model [9]. Two decades calculations show that, the global color symmetry model (GCM) [10] describes the nonperturbative aspects of strong interaction physics and hadronic phenomena at low energies quite well ([11]-[29]), we naturally expect that GCM is applicable in the estimation of the tensor susceptibility of QCD vacuum. Recent investigation shows that, the value of tensor susceptibility calculated from an effective quark-quark interaction is much smaller than the others [30]. Questions are therefore arising. What is the reason for this discrepancy? Is anything wrong with GCM? This letter aims to answer these questions.

The QCD partition function for massless quarks in Euclidean space can be written as

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}Ae^{-S[\bar{q},q,A]} \quad (1)$$

with the QCD action

$$S[\bar{q}, q, A] = \int dx [\bar{q}(x)(\not{\partial} - ig\not{A})q(x) + \frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a], \quad (2)$$

where  $A_\mu = A_\mu^a \frac{\lambda^a}{2}$ ,  $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ . By introducing the functional  $W[J]$  defined as

$$e^{W[J]} \equiv \int \mathcal{D}A \exp \left( \int dx \left( -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + J_\mu^a A_\mu^a \right) \right), \quad (3)$$

the QCD partition function can be rewritten as

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q e^{-\int dx \bar{q}(x)\not{\partial}q(x)} e^{W[ig\bar{q}\gamma_\mu \frac{\lambda^a}{2}q]}. \quad (4)$$

The functional  $W[J]$  has the expansion

$$W[J] = \frac{1}{2} \int dx dy J_\mu^a(x) D_{\mu\nu}^{ab}(x, y) J_\nu^b(y) + W_R[J], \quad (5)$$

where  $D_{\mu\nu}^{ab}(x, y) = D_{\mu\nu}^{ab}(x - y)$  is the gluon 2-point Green's function, and  $W_R[J]$  involves the higher order  $n(\geq 3)$ -point Green's functions. The GCM

is obtained through the truncation of the functional  $W[J]$  in which only  $D_{\mu\nu}^{ab}(x, y)$  is retained. This model maintains global color symmetry of QCD, but the local color SU(3) gauge invariance is lost. For simplicity we use a Feynman-like gauge  $D_{\mu\nu}^{ab}(x - y) = \delta_{\mu\nu} \delta^{ab} D(x - y)$ . The important dynamical characteristics of local color symmetry are included in  $D(x)$ . The exact form of  $D(x)$  is not well known. Instead, we use a phenomenological gluon propagator, which is required to exhibit the properties of asymptotic freedom and infrared slavery. The justification of such truncation relies on the successes of various calculations.

The partition function of this truncation can be given as [10, 31]

$$\mathcal{Z}_{\text{GCM}} = \int \mathcal{D}\bar{q} \mathcal{D}q \exp \left( - \int dx \bar{q} \not{\partial} q - \frac{g^2}{2} \int dx dy j_\mu^a(x) D_{\mu\nu}^{ab}(x - y) j_\nu^b(y) \right), \quad (6)$$

with the quark color current  $j_\mu^a(x) = \bar{q}(x) \gamma_\mu \frac{\lambda^a}{2} q(x)$ , or equivalently,

$$\mathcal{Z}_{\text{GCM}} = \int \mathcal{D}\bar{q} \mathcal{D}q \mathcal{D}A e^{-S_{\text{GCM}}[\bar{q}, q, A]} \quad (7)$$

with the GCM action

$$S_{\text{GCM}}[\bar{q}, q, A] = \int dx [\bar{q}(x) (\not{\partial} - ig \not{A}) q(x) + \int dx dy \frac{1}{2} A_\mu^a(x) [D_{\mu\nu}^{ab}(x - y)]^{-1} A_\nu^b(y)]. \quad (8)$$

By the standard bosonization procedure, the resulting expression for the partition function in terms of the bilocal field integration is

$$\mathcal{Z}_{\text{GCM}} = \int \mathcal{D}\mathcal{B}^\theta \exp \left( -S[\mathcal{B}^\theta] \right), \quad (9)$$

where the action is given by

$$S[\mathcal{B}^\theta] = -\text{Tr} \text{Ln}[G^{-1}] + \int dx dy \frac{\mathcal{B}^\theta(x, y) \mathcal{B}^\theta(y, x)}{2g^2 D(x - y)}, \quad (10)$$

and the quark inverse Green's function  $G^{-1}$  is defined as

$$G^{-1}(x, y) = \not{\partial} \delta(x - y) + \Lambda^\theta \mathcal{B}^\theta(x, y). \quad (11)$$

Here the quantity  $\Lambda^\theta$  arises from Fierz reordering of the current-current interaction term in Eq. (6)

$$\Lambda_{ji}^\theta \Lambda_{lk}^\theta = (\gamma_\mu \frac{\lambda^a}{2})_{jk} (\gamma_\mu \frac{\lambda^a}{2})_{li} \quad (12)$$

and is the direct product of Dirac, flavor SU(2) and color matrices:

$$\Lambda^\theta = \frac{1}{2}(I_D, i\gamma_5, \frac{i}{\sqrt{2}}\gamma_\mu, \frac{i}{\sqrt{2}}\gamma_\mu\gamma_5) \otimes (\frac{1}{\sqrt{2}}I_F, \frac{1}{\sqrt{2}}\vec{\tau}_F) \otimes (\frac{4}{3}I_c, \frac{i}{\sqrt{3}}\lambda_c^a). \quad (13)$$

Here we consider  $N_F = 2$  flavors as in Ref. [9].

The vacuum configurations are defined by minimizing the bilocal action:  $\frac{\delta S[\mathcal{B}]}{\delta B}\Big|_{\mathcal{B}_0} = 0$ , which gives

$$\mathcal{B}_0^\theta(x) = g^2 D(x) \text{tr}[G_0(x) \Lambda^\theta]. \quad (14)$$

These configurations provide self-energy dressing of the quarks through the definition  $\Sigma(p) \equiv \Lambda^\theta \mathcal{B}_0^\theta(p) = i\not{p}[A(p^2) - 1] + B(p^2)$ . According to Ref. [10], the self-energy functions  $A$  and  $B$  satisfy the Dyson-Schwinger equations,

$$[A(p^2) - 1]p^2 = \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} g^2 D((p-q)^2) \frac{A(q^2)q \cdot p}{q^2 A^2(q^2) + B^2(q^2)}, \quad (15)$$

$$B(p^2) = \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} g^2 D((p-q)^2) \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \quad (16)$$

The quark Green's function at  $\mathcal{B}_0^\theta$  is given by

$$G_0(x, y) = G_0(x - y) = \int \frac{d^4 p}{(2\pi)^4} \frac{-i\not{p}A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} e^{ip \cdot (x-y)}. \quad (17)$$

The hadron properties follow from considering deviations from these vacuum configurations. If we consider only the isoscalar  $\sigma(x)$  and isovector  $\vec{\pi}(x)$  fields, the approximate local-field effective action can be taken as [10]

$$\begin{aligned} S[\sigma, \vec{\pi}] &= -\text{Tr} \text{Ln} \left\{ \not{\partial} A(x-y) + m\delta^{(4)}(x-y) + V[\sigma, \vec{\pi}]B(x-y) \right\} \\ &+ \frac{1}{2} \int d^4 z [\sigma^2(z) + \vec{\pi}^2(z)] \int d^4 w B(w) \text{tr}[G(w)], \end{aligned} \quad (18)$$

where  $V[\sigma, \vec{\pi}] = \sigma(\frac{x+y}{2}) + i\gamma_5 \vec{\pi}(\frac{x+y}{2}) \cdot \vec{\tau}$  and  $m$  is the quark bare mass. Expanding the spectrum of  $S[\sigma, \vec{\pi}]$  to second order about its minimum  $S[1, 0]$  with  $\sigma(x) = 1 + \delta(x)$

$$S[1+\delta(x), \vec{\pi}(x)] - S[1, 0] = \frac{1}{2} f_\delta^2 \int [(\partial_\mu \delta)^2 + m_\delta^2 \delta^2] d^4 z + \frac{1}{2} f_\pi^2 \int [(\partial_\mu \vec{\pi})^2 + m_\pi^2 \vec{\pi}^2] d^4 z + \dots, \quad (19)$$

it is found that

$$m_\delta^2 = \frac{3}{2\pi^2 f_\delta^2} \int_0^\infty s ds \frac{B^2(s)[B^2(s) - sA^2(s)]}{[sA^2(s) + B^2(s)]^2}, \quad (20)$$

$$f_\delta^2 = \frac{3}{8\pi^2} \int_0^\infty s ds \frac{A^2(s)B^2(s)}{[sA^2(s) + B^2(s)]^3} \left\{ 2sA^2(s) + \frac{B^2(s)[sA^2(s) - B^2(s)]}{sA^2(s) + B^2(s)} \right\}; \quad (21)$$

and

$$m_\pi^2 = \frac{3m}{2\pi^2 f_\pi^2} \int_0^\infty s ds \frac{B(s)}{sA^2(s) + B^2(s)}, \quad (22)$$

$$f_\pi^2 = \frac{3}{8\pi^2} \int_0^\infty s ds \frac{A^2(s)B^2(s)}{[sA^2(s) + B^2(s)]^2} \left[ 2 + \frac{B^2(s)}{sA^2(s) + B^2(s)} \right]. \quad (23)$$

In Eqs. (21) and (23), all those terms involving the derivatives of  $A(s)$  and  $B(s)$  with respect to  $s$  are neglected. Let  $A(s) = 1$ ,  $B(s) = M$ , with  $M$  the mass of the constituent quark, Eq. (23) reduces to

$$f_\pi^2 = \frac{3}{8\pi^2} \int_0^\infty s ds \frac{M^2}{(s + M^2)^2} \left( 2 + \frac{M^2}{s + M^2} \right). \quad (24)$$

When  $s$  approaches infinity, the integrand of Eq. (24) behaves like  $2M^2/s$ , which reproduce the result of Ref. [9] strictly. While  $s \rightarrow 0$ , it behaves like  $3s/M^2$  rather than  $2s/M^2$  of Ref. [9]. This difference arises from the second term in the brackets of Eq. (24). Can this difference bring some serious problems? Let put this question aside at present.

The tensor susceptibility  $\chi$  is defined as [5]

$$\chi \equiv \frac{\Pi_\chi(0)}{6\langle\bar{q}q\rangle}, \quad (25)$$

where

$$\langle\bar{q}q\rangle = -\frac{3}{4\pi^2} \int_0^\infty s ds \frac{B(s)}{sA^2(s) + B^2(s)} \quad (26)$$

is the quark condensate, we need only to calculate  $\Pi_\chi(0)$  defined as [30]:

$$\frac{1}{12}\Pi_\chi(0) \equiv -\frac{3}{4\pi^2} \int_0^\infty s ds \left[ \frac{B(s)}{sA^2(s) + B^2(s)} \right]^2. \quad (27)$$

In this equation, if we let  $A(s) = 1$ ,  $B(s) = M$ , the result of Ref. [9] is strictly reproduced.

Before the numerical calculation of tensor susceptibility, we make an analysis of the nontrivial solutions  $A(s)$  and  $B(s)$  to the Dyson-Schwinger equations (15) and (16). Because the phenomenological gluon propagators exhibit the properties of asymptotic freedom and infrared slavery, when  $s = p^2$  increases from 0 to  $\infty$ ,  $B(s)$  ( $\geq 0$ ) decreases from some finite non-zero value to zero, while  $A(s)$  decreases from some non-zero value down to 1. Therefore,  $A(s)$  is universally greater than 1. The replacement of  $A(s)$  by 1 in Eq. (27) will lead an increase in the value of  $\Pi_\chi(0)/12$ , or in other words, the value calculated from Eq. (27) is expected to be smaller than that obtained by the others. This is verified in the recent calculation [30], and will be further checked in the calculation below.

The tactics in calculation is similar to that of Ref. [30]. The input parameters are adjusted to reproduce the pion decay constant in the chiral limit  $f_\pi = 87$  MeV via Eq. (23).

To give a convincing conclusion, we choose three different gluon propagators. The ultraviolet behavior of these model gluon propagators are different from that in QCD [32, 33]. They are model 1:

$$g^2 D(s) = 4\pi^2 d \frac{\lambda^2}{s^2 + \Delta}, \quad (28)$$

model 2:

$$g^2 D(s) = 3\pi^2 \frac{\lambda^2}{\Delta^2} e^{-\frac{s}{\Delta}} + \frac{4\pi^2 d}{s \ln[s/\Lambda^2 + e]}, \quad (29)$$

and model 3:

$$g^2 D(s) = 4\pi^2 d \frac{\lambda^2}{s^2 + \Delta} + \frac{4\pi^2 d}{s \ln[s/\Lambda^2 + e]}. \quad (30)$$

Here  $d = 12/(33 - 2N_f) = 12/29$  and  $\Lambda = 200$  MeV. For model 1, the self-energy functions  $A(s)$  and  $B(s)$  varying with  $s$  are showed respectively in Figs. 1 and 2, with the input parameters  $\Delta = 0.1 \text{ GeV}^4$ ,  $\lambda = 1.780 \text{ GeV}$ . Obviously,  $A(s)$  is not less than 1. In Table 1 the values of  $\Pi_\chi(0)/12$  for model 1 are displayed, and the corresponding values for quark condensate  $\langle \bar{q}q \rangle$  are also listed. It should be noted that the values for quark condensate are roughly around those obtained from QCD sum rules (see Ref. [31] and references therein), due to the term  $B(s)/(sA^2(s) + B^2(s))$  in Eq. (26) against the term  $[B(s)/(sA^2(s) + B^2(s))]^2$  in Eq. (27).

The results for model 2 and model 3 are given in Table 2. It is shown that the values of the quantity  $\Pi_\chi(0)/12$  are still very small as that of Ref. [30].

Table 1: The values of  $\Pi_\chi(0)/12$  for model 1 with Eq. (23) used to calculate  $f_\pi$ . The quark condensate  $\langle\bar{q}q\rangle$  is also presented.

$\Delta$ [GeV <sup>4</sup> ]	$\lambda$ [GeV]	$-\langle\bar{q}q\rangle^{1/3}$ [MeV]	$\Pi_\chi(0)/12$ [GeV <sup>2</sup> ]
$10^{-1}$	1.780	279	-0.0017
$10^{-2}$	1.350	244	-0.0015
$10^{-4}$	0.955	210	-0.0013
$10^{-6}$	0.770	196	-0.0013

Table 2: The numerical results for models 2 and 3 with Eq. (23) used to calculate  $f_\pi$ .

model 2			model 3		
$\Delta$ [GeV <sup>2</sup> ]	$\lambda$ [GeV]	$\Pi_\chi(0)/12$ [GeV <sup>2</sup> ]	$\Delta$ [GeV <sup>4</sup> ]	$\lambda$ [GeV]	$\Pi_\chi(0)/12$ [GeV <sup>2</sup> ]
2.000	2.94	-0.0021	$10^{-1}$	1.71	-0.0016
0.200	1.51	-0.0015	$10^{-4}$	0.95	-0.0012
0.020	1.44	-0.0012	$10^{-7}$	0.71	-0.0012

To further check if the second term in the brackets of Eq. (23) give some serious modifications to our results, we drop this term intentionally and obtain:

$$f_\pi^2 = \frac{3}{4\pi^2} \int_0^\infty s ds \frac{A^2(s)B^2(s)}{[sA^2(s) + B^2(s)]^2}. \quad (31)$$

Let  $A(s) = 1$ ,  $B(s) = M$ , the result of Ref. [9] is strictly reproduced. With Eq. (31), similar calculations can be performed for the various models as previously did. For example, for models 2 and 3, the results are given in Table 3. One can see that, these results do not make much difference to the previous ones.

To summarize, we have calculated the QCD vacuum tensor susceptibility based on the modified version of calculating pion decay constant. The various calculations in this letter and in Ref. [30] show that, it is a fact that the value of tensor susceptibility calculated in GCM is very small. In these calculations, the basic characteristic ( $A(s) \geq 1$ ) of quark propagator  $G_0(p)$  or quark self-energy  $\Sigma(p)$  determined from the Dyson-Schwinger equations keeps

Table 3: The numerical results for models 2 and 3 with Eq. (31) used to calculate  $f_\pi$ .

model 2			model 3		
$\Delta$ [GeV <sup>2</sup> ]	$\lambda$ [GeV]	$\Pi_\chi(0)/12$ [GeV <sup>2</sup> ]	$\Delta$ [GeV <sup>4</sup> ]	$\lambda$ [GeV]	$\Pi_\chi(0)/12$ [GeV <sup>2</sup> ]
2.000	2.955	-0.0023	$10^{-1}$	1.755	-0.0018
0.200	1.600	-0.0017	$10^{-4}$	1.010	-0.0014
0.020	1.570	-0.0015	$10^{-7}$	0.765	-0.0014

unchanged. This is probably the main reason to the small value of tensor susceptibility. So, for the calculation of QCD vacuum tensor susceptibility, GCM formulated to date deviates seriously from the QCD sum rules and constituent quark model.

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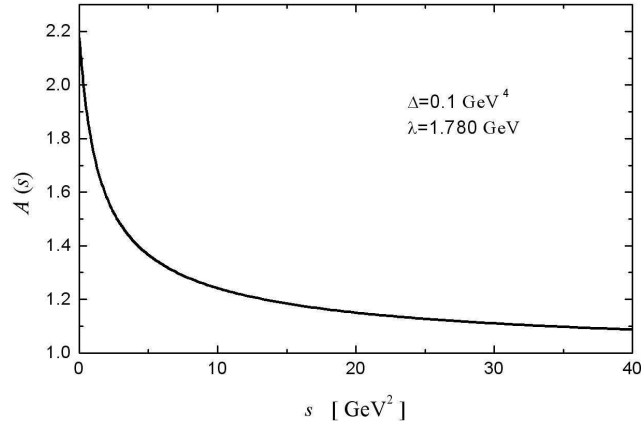


Figure 1: The self-energy function  $A(s)$  as a function of  $s$  for the gluon propagator  $g^2 D(s) = \frac{48\pi^2}{29} \frac{\lambda^2}{s^2 + \Delta}$ , with  $\Delta = 0.1 \text{ GeV}^4$ ,  $\lambda = 1.780 \text{ GeV}$

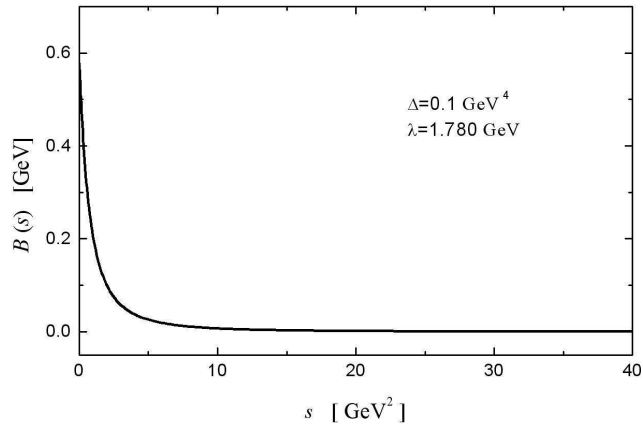


Figure 2: The self-energy function  $B(s)$  as a function of  $s$  for the gluon propagator  $g^2 D(s) = \frac{48\pi^2}{29} \frac{\lambda^2}{s^2 + \Delta}$ , with  $\Delta = 0.1 \text{ GeV}^4$ ,  $\lambda = 1.780 \text{ GeV}$